IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

Almost Contra-β-Open and Almost Contra-β-Closed Mappings

Dr. Balasubramanian.S. [*1], C. Sandhya[2]

Assistant Professor, Dept. of Mathematics, Government Arts College(A), Karur – 639 005, Tamilnadu, India

S.G.Lecturer, Depatment of Mathematics, C.S.R. Sarma College, Ongole – 523 002, Andhrapradaesh, India

sandhya_karavadi@yahoo.co.uk

Abstract

The aim of this paper is to introduce and study the concepts of almost contra-β-open and almost contra-β-closed mappings.

Keywords: Open set, Closed set, Open map, Closed map, Contra-β-Open Map, Contra-β-Closed Map, Almost Contra-β-Open Map and Almost Contra-β-Closed Map.

AMS Classification: 54C10, 54C08, 54C05.

Introduction

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Open and closed mappings are one such mappings which are studied for different types of open and closed sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1970, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced α-open and α-closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-clsoed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced β-open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about β-closed mappings in the year 2000. C. W. Baker, introduced Contra-open functions and contra-closed functions in the year 1997. M.Caldas and C.W.Baker introduced contra pre-semiopen Maps in the year 2000. Inspired with these concepts and its interesting properties we in this paper tried to study a new variety of open and closed maps called almost contra-β-open and almost contra-β-closed maps. Throughout the paper X, Y means topological spaces (X, τ) and (Y, σ) on which no separation axioms are assured.

Preliminaries

Definition 2.1: $A \subseteq X$ is said to be

a) regular open[pre-open; semi-open; α-open; β-open] if $A = int(cl(A))$ [$A \subseteq int(cl(A))$; $A \subseteq cl(int(A))$; $A \subseteq int(cl(int(A)))$; $A \subseteq cl(int(cl(A)))$] and regular closed[pre-closed; semi-closed; α-closed; β-closed] if $A = cl(int(A))$[$cl(int(A)) \subseteq A$; $int(cl(A)) \subseteq A$; $cl(int(cl(A))) \subseteq A$; $int(cl(int(A))) \subseteq A$]

b) g-closed[rg-closed] if $cl(A) \subset U$[$rcl(A) \subset U$] whenever $A \subset U$ and U is open[r-open] in X and g-open[rg-open] if its complement X - A is g-closed[rg-closed].

Definition 2.2: A function $f: X \to Y$ is said to be

a) continuous[resp: semi-continuous, r-continuous] if the inverse image of every open set is open [resp: semi open, regular open] and g-continuous [resp: rg-continuous] if the inverse image of every closed set is g-closed. [resp: rg-closed].
b) irresolute [resp: r-irresolute] if the inverse image of every semi open [resp: regular open] set is semi open. [resp: regular open].
c) open[resp: semi-open; pre-open] if the image of every open set in X is open[resp: semi-open; pre-open] in Y.

d) closed[resp: semi-closed, r-closed] if the image of every closed set is closed [resp: semi closed, regular closed].
e) contra-open[resp: contra semi-open; contra pre-open] if the image of every open set in *X* is closed[resp: semi-closed; pre-closed] in *Y*.
f) contra closed[resp: contra semi-closed; contra pre-closed] if the image of every closed set in *X* is open[resp: semi-open; pre-open] in *Y*.

Remark 1: We have the following implication diagrams for open sets and closed sets.

```
          semi-open →  → β-open                 semi-closed →  → β-closed
              ↑              ↑                        ↑              ↑
r-open → open → α-open → pre-open    r-closed → closed → α-closed → pre-closed
```

Definition 2.3: *X* is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

Almost Contra β-Open Mappings

Definition 3.1: A function *f: X*→*Y* is said to be almost contra β-open if the image of every r-open set in *X* is β-closed in *Y*.

Example 1: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, *X*}; σ = {ϕ, {a}, {b}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = b, *f*(b) = c and *f*(c) = a. Then *f* is almost contra-*β*-open and contra-*β*-open.

Example 2: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, *X*}; σ = {ϕ, {a}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = a, *f*(b) = c and *f*(c) = b. Then *f* is not almost contra-*β*-open and contra-*β*-open

Theorem 3.1: Every contra-*β*-open map is almost contra-*β*-open but not conversely.
Proof: Let A⊆*X* be r-open ⇒ A is open ⇒ *f*(A) is *β*-closed in *Y*. since *f*:*X*→*Y* is contra-*β*-open. Hence *f* is almost contra-*β*-open.

Example 3: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, *X*}; σ = {ϕ, {a}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = a, *f*(b) = c and *f*(c) = b. Then *f* is almost contra-*β*-open but not contra-*β*-open.

Note 1: We have the following implication diagram among the open maps.

```
                    c.s.o. →  → → → → →  c.β.o
                      ↓                    ↓
                    al.c.s.o. →  →  → → →  al.c.β.o
                      ↑                    ↑
Al.c.r.o → → → al.c.o. → al.c.α.o. → al.c.p.o.
                      ↑         ↑          ↑
c.r.o → → → → c.o. → →   c.α.o. → → c.p.o.          None is reversible.
```

Example 4: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, *X*}; σ = {ϕ, {a}, {b}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = b, *f*(b) = c and *f*(c) = a. Then *f* is almost contra-*β*-open, almost contra-semi-open, contra-*β*-open and contra semi-open but not almost contra-open, almost contra-pre-open, contra-open and contra-pre-open.

Example 5: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, *X*}; σ = {ϕ, {a, c}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = b, *f*(b) = c and *f*(c) = a. Then *f* is almost contra-*β*-open, almost contra pre-open, contra-*β*-open and contra-pre-open but not almost contra-open, almost contra-semi-open, contra-open and contra-semi-open.

Example 6: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, *X*}; σ = {ϕ, {a}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = a, *f*(b) = c and *f*(c) = b. Then *f* is almost contra-open, almost contra-semi-open, almost contra-pre-open and almost contra-*β*-open, but not contra-open, contra-semi-open, contra-pre-open and contra-*β*-open.

Example 7: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, *X*}; σ = {ϕ, {a}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = a, *f*(b) = c and *f*(c) = b. Then *f* is not almost contra-open, almost contra-semi-open, almost contra-pre-open, almost contra-*β*-open, contra-open, contra-semi-open, contra-pre-open and contra-*β*-open.

Note 2: If βC(Y) = RC(Y) We have the following implication diagram among the open maps.

al.c.s.o. ↔ ↔ ↔ ↔ ↔ al.c.β.o
↑↓ ↑↓
Al.c.r.o ↔ ↔ al.c.o. ↔ al.c.α.o. ↔ al.c.p.o.

Theorem 3.1: If *f* is open[r-open] and *g* is contra β-open then *g o f* is almost contra β-open.
Proof: Let A⊆*X* be r-open ⇒ *f*(A) is open[r-open] in *Y* ⇒ *g*(*f*(A)) = *gof*(A) is β-closed in *Z*. Hence *g o f* is almost contra β-open.

Theorem 3.2: If *f* is almost contra open[almost contra-r-open] and *g* is β-closed then *g o f* is almost contra-β-open.
Proof: Let A⊆*X* be r-open in X ⇒ *f*(A) is closed[r-closed] in Y ⇒ *g*(*f*(A)) = *g*•*f*(A) is β-closed in Z. Hence *g*• *f* is almost contra β-open.

Corollary 3.1:
(i) If *f* is open[r-open] and *g* is contra *r*-open then *g o f* is almost contra β-open.
(ii) If *f* is almost contra open[almost contra-r-open] and *g* is *r*-closed then *g o f* is almost contra-β-open.

Theorem 3.3: If *f: X*→*Y* is almost contra β-open then $\beta\overline{(f(A))} \subseteq f(\bar{A})$
Proof: Let A⊆*X* and *f: X*→*Y* be almost contra β-open. Then $f(\bar{A})$ is β-closed in *Y* and $f(A) \subseteq f(\bar{A})$. This implies $\beta\overline{(f(A))} \subseteq \beta\overline{(f(\bar{A}))}$ → (1)

Since $f(\bar{A})$ is β-open in *Y*, $\beta\overline{(f(\bar{A}))} = f(\bar{A})$ → (2)

Using (1) & (2) we have $\beta\overline{(f(A))} = f(\bar{A})$ for every subset A of *X*.

Remark 2: Converse is not true in general.

Example 8: Let *X* = *Y* = {a, b, c} τ = {ϕ, {a}, {a, b}, *X*} σ = {ϕ, {a}, {b}, {a, b}, *Y*}. Let *f: X*→*Y* be the identity map. Then $\beta\overline{(f(A))} \subseteq f(\bar{A})$ for every subset A of *X*. But *f* is not contra β-open since *f*({a, b}) = {a, b} is not β-closed.

Remark 3: Similarly one can verify the case for almost contra β-open.

Corollary 3.2: If *f*:*X*→*Y* is almost contra *r*-open then $\beta\overline{(f(A))} \subseteq f(\bar{A})$.

Theorem 3.4: If *f*:*X*→*Y* is almost contra β-open and A⊆*X* is open, *f*(A) is τ_{β}-closed in *Y*.
Proof: Let A⊆*X* and *f*:*X*→*Y* be almost contra β-open ⇒ $\beta\overline{(f(A))} \subseteq f(\bar{A})$ (by theorem 3.3.) ⇒ $\beta\overline{(f(A))} \subseteq f(A)$ since $f(A) = f(\bar{A})$ as A is open. But $f(A) \subseteq \beta\overline{(f(A))}$. Therefore we have $f(A) = \beta\overline{(f(A))}$. Hence *f*(A) is τ_{β}-closed in *Y*.

Corollary 3.3: If *f*:*X*→*Y* is almost contra *r*-open, then *f*(A) is τ_{β}-closed in *Y* if A is *r*-open set in *X*.

Theorem 3.5: *f*:*X*→*Y* is almost contra β-open iff for each subset S of *Y* and each r-closed set U containing $f^{-1}(S)$, there is an β-open set V of *Y* such that S⊆ V and $f^{-1}(V) \subseteq U$.

Remark 4: Composition of two almost contra β-open maps is not almost contra β-open in general.

Theorem 3.6: Let *X*, *Y*, *Z* be topological spaces and every β-closed set is *r*-open in *Y*. Then the composition of two almost contra β-open[almost contra *r*-open] maps is almost contra β-open.
Proof: (a) Let *f*:*X*→*Y* and g:*Y*→*Z* be almost contra β-open maps. Let A be any r-open set in *X* ⇒ *f*(A) is β-closed in *Y*⇒ *f*(A) is r-open in *Y* (by assumption) ⇒ *g*(*f*(A)) = *gof*(A) is β-closed in *Z*. Therefore *g o f* is almost contra β-open.

(b) Let *f*:*X*→*Y* and g:*Y*→*Z* be almost contra β-open maps. Let A be any r-open set in *X* ⇒ *f* (A) is *r*-closed in *Y* ⇒ *f*(A) is β-closed in *Y*⇒ *f* (A) is r-open in *Y* (by assumption) ⇒ *g*(*f* (A)) is *r*-closed in *Z* ⇒ *g o f*(A) is β-closed in *Z*. Therefore *gof* is almost contra β-open.

Theorem 3.7: Let *X*, *Y*, *Z* be topological spaces and Y is discrete topological space in *Y*. Then the composition of two almost contra β-open[almost contra *r*-open] maps is almost contra β-open.

Theorem 3.8: If f:$X\rightarrow Y$ is g-open, g:$Y\rightarrow Z$ is almost contra β-open [almost contra r-open] and Y is $T_{1/2}$ [r-$T_{1/2}$] then g o f is almost contra β-open.
Proof: (a) Let A be an r-open set in X. Then f(A) is g-open set in $Y \Rightarrow f$ (A) is open in Y as Y is $T_{1/2} \Rightarrow g(f(A)) = gof(A)$ is β-closed in Z since g is almost contra β-open. Hence gof is almost contra β-open.

(b) Let A be an r-open set in X. Then f(A) is g-open set in $Y \Rightarrow f$ (A) is open in Y as Y is $T_{1/2} \Rightarrow g(f(A)) = gof(A)$ is r-closed in Z since g is almost contra r-open $\Rightarrow gof(A)$ is β-closed in Z. Hence gof is almost contra β-open.

Theorem 3.9: If f:$X\rightarrow Y$ is rg-closed, g:$Y\rightarrow Z$ is almost contra β-open [almost contra r-open] and Y is r-$T_{1/2}$, then g o f is almost contra β-open.
Proof: Let A be an r-open set in X. Then f(A) is rg-open in $Y \Rightarrow f$(A) is r-open in Y since Y is r-$T_{1/2} \Rightarrow$ g(f(A)) = go$f$(A) is β-closed in Z. Hence gof is almost contra β-open.

Theorem 3.10: If f:$X\rightarrow Y$, g:$Y\rightarrow Z$ be two mappings such that gof is almost contra β-open [almost contra r-open] then the following statements are true.
a) If f is continuous [r-continuous] and surjective then g is almost contra β-open.
b) If f is g-continuous, surjective and X is $T_{1/2}$ then g is almost contra β-open.
c) If f is rg-continuous, surjective and X is r-$T_{1/2}$ then g is almost contra β-open.
Proof: (a) Let A be an r-open set in $Y \Rightarrow f^{-1}$(A) is open in $X \Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ is β-closed in Z. Hence g is almost contra β-open.

(b) Let A be an r-open set in $Y \Rightarrow f^{-1}$(A) is g-open in $X \Rightarrow f^{-1}$(A) is open in X[since X is $T_{1/2}$] $\Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ is β-closed in Z. Hence g is almost contra β-open.

(c) Let A be an r-open set in Y. $\Rightarrow f^{-1}$(A) is g-open in $X \Rightarrow f^{-1}$(A) is open in X[since X is r-$T_{1/2}$] $\Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ is β-closed in Z. Hence g is almost contra β-open.

Theorem 3.11: If f:$X\rightarrow Y$ is almost contra β-open and A is an open set of X then f_A:(X, τ(A)) → (Y, σ) is almost contra β-open.
Proof: (a) Let F be an r-open set in A. Then F = A∩E for some open set E of X and so F is open in $X \Rightarrow f$(F) is β-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is almost contra β-open.

Theorem 3.12: If f:$X\rightarrow Y$ is almost contra β-open, X is $T_{1/2}$ and A is g-open set of X then f_A:(X, τ(A)) → (Y, σ) is almost contra β-open.
Proof: Let F be an r-open set in A. Then F = A∩E for some open set E of X and so F is open in $X \Rightarrow f$(F) is β-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is almost contra β-open.

Corollary 3.4: If f:$X\rightarrow Y$ is almost contra r-open
(i) and A is an open set of X then f_A:(X, τ(A)) → (Y, σ) is almost contra β-open.
(ii) X is $T_{1/2}$ and A is g-open set of X then f_A:(X, τ(A)) → (Y, σ) is almost contra β-open.

Theorem 3.13: If f_i: $X_i \rightarrow Y_i$ be almost contra β-open [almost contra r-open] for i =1, 2. Let f: X_1x$X_2 \rightarrow Y_1$xY_2 be defined as $f(x_1,x_2) = (f_1(x_1),f_2(x_2))$. Then f:X_1x$X_2 \rightarrow Y_1$xY_2 is almost contra β-open.
Proof: Let U_1x$U_2 \subseteq X_1$xX_2 where U_i is r-open in X_i for i = 1,2. Then $f(U_1$x$U_2) = f_1(U_1)$ x $f_2(U_2)$ is β-closed set in Y_1xY_2. Hence f is almost contra β-open.

Theorem 3.14: Let h:$X\rightarrow X_1$xX_2 be almost contra β-open. Let f_i:$X\rightarrow X_i$ be defined as $h(x)=(x_1,x_2)$ and $f_i(x) = x_i$. Then f_i: $X\rightarrow X_i$ is almost contra β-open for i =1, 2.
Proof: Let U_1 be r-open in X_1, then $U_1 x\ X_2$ is r-open in $X_1 x\ X_2$, and $h(U_1 x\ X_2)$ is β-closed in X. But $f_1(U_1) = h(U_1 x\ X_2)$, therefore f_1 is almost contra β-open. Similarly we can show that f_2 is also almost contra β-open and thus f_i: X → X_i is almost contra β-open for i = 1, 2.

Almost Contra β-Closed Mappings

Definition 4.1: A function *f: X*→*Y* is said to be almost contra β-closed if the image of every r-closed set in *X* is β-open in *Y*.

Example 9: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, *X*}; σ = {ϕ, {a}, {b}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = b, *f*(b) = c and *f*(c) = a. Then *f* is almost contra-*β*-closed and contra-*β*-closed.

Example 10: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, *X*}; σ = {ϕ, {a}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = a, *f*(b) = c and *f*(c) = b. Then *f* is not almost contra-*β*-closed and contra-*β*-closed.

Theorem 3.1: Every contra-*β*-closed map is almost contra-*β*-closed but not conversely.
Proof: Let A⊆*X* be r-closed ⇒ A is closed ⇒ *f*(A) is *β*-open in *Y*. since *f*:*X*→*Y* is contra-*β*-closed. Hence *f* is almost contra-*β*-closed.

Example 11: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, *X*}; σ = {ϕ, {a}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = a, *f*(b) = c and *f*(c) = b. Then *f* is almost contra-*β*-closed but not contra-*β*-closed.

Note 3: We have the following implication diagram among the closed maps.

c.s.c. → → → → → → c.β.c
↓ ↓
al.c.s.c. → → → → → al.c.β.c
↑ ↑
Al.c.r.c → → → al.c.c. → al.c.α.c. → al.c.p.c.
↑ ↑ ↑
c.r.c → → → → c.c. → → c.α.c. → → c.p.c. None is reversible.

Example 12: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, *X*}; σ = {ϕ, {a}, {b}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = b, *f*(b) = c and *f*(c) = a. Then *f* is almost contra-*β*-closed, almost contra-semi-closed, contra-*β*-closed and contra semi-closed but not almost contra-closed, almost contra-pre-closed, contra-closed and contra-pre-closed.

Example 13: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, *X*}; σ = {ϕ, {a, c}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = b, *f*(b) = c and *f*(c) = a. Then *f* is almost contra-*β*-closed, almost contra pre-closed, contra-*β*-closed and contra-pre-closed but not almost contra-closed, almost contra-semi-closed, contra-closed and contra-semi-closed.

Example 14: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, *X*}; σ = {ϕ, {a}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = a, *f*(b) = c and *f*(c) = b. Then *f* is almost contra-closed, almost contra-semi-closed, almost contra-pre-closed and almost contra-*β*-closed, but not contra-closed, contra-semi-closed, contra-pre-closed and contra-*β*-closed.

Example 15: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, *X*}; σ = {ϕ, {a}, {a, b}, *Y*}. Let *f*:*X*→*Y* be defined *f*(a) = a, *f*(b) = c and *f*(c) = b. Then *f* is not almost contra-closed, almost contra-semi-closed, almost contra-pre-closed, almost contra-*β*-closed, contra-closed, contra-semi-closed, contra-pre-closed and contra-*β*-closed.

Note 4: If βO(Y) = RO(Y) following implication diagram among closed maps is true.

al.c.s.c. ↔ ↔ ↔ ↔ ↔ al.c.β.c
↑↓ ↑↓
Al.c.r.c ↔ ↔ al.c.c. ↔ al.c.α.c. ↔ al.c.p.c.

Theorem 4.1: If *f* is closed[r-closed] and *g* is contra β-closed then *gof* is almost contra β-closed.

Proof: Let A⊆*X* be r-closed⇒ *f*(A) is closed[r-closed] in *Y* ⇒ *g*(*f*(A)) is β-open in *Z* ⇒ *g o f*(A) is β-open in *Z*. Hence *g o f* is almost contra β-closed.

Theorem 4.2: If f is almost contra closed[almost contra-r-closed] and g is β-open then $g\ o\ f$ is almost contra-β-closed.

Proof: Let A⊆X be r-closed in X ⇒ f(A) is open[r-open] in Y ⇒ $g(f(A)) = g \bullet f(A)$ is β-open in Z. Hence $g \bullet f$ is almost contra β-closed.

Corollary 4.1:
(i) If f is closed[r-closed] and g is contra closed[contra-r-closed, contra semi-closed, contra pre-closed] then $g\ o\ f$ is almost contra β-closed.
(ii) If f is almost contra closed[almost contra-r-closed, contra semi-closed, contra pre-closed] and g is r-open then $g\ o\ f$ is almost contra-β-closed.

Theorem 4.3: If $f: X\to Y$ is almost contra β-closed, then $f(A^o)\subset \beta(f(A))^o$

Proof: Let A⊆ X be r-closed and f: X→ Y is almost contra β-closed gives $f(A^o)$ is β-open in Y and $f(A^o)\subset f(A)$ which in turn gives $\beta(f(A^o))^o\subset \beta(f(A))^o$---(1)
Since $f(A^o)$ is β-open in Y, $\beta(f(A^o))^o = f(A^o)$------------------(2)
combining (1) and (2) we have $f(A^o)\subset \beta(f(A))^o$ for every subset A of X.

Remark 5: Converse is not true in general as shown by the following example.

Example 16: Let $X = Y$ = {a, b, c} τ = {ϕ, {a}, {a, b}, X} σ = {ϕ, {a}, {b}, {a, b}, Y}. Let $f: X\to Y$ be the identity map. Then $f(A^o)\subset \beta(f(A))^o$ for every subset A of X. But f is not contra β-closed since f({c}) = {c} is not β-open.

Remark 6: Similarly one can verify the case for almost contra β-open.

Corollary 4.2: If $f:X\to Y$ is almost contra r-closed, then $f(A^o)\subset \beta(f(A))^o$

Proof: Let A⊆ X be r-closed and f: X→ Y is almost contra r-closed gives $f(A^o)$ is r-open in Y and $f(A^o)\subset f(A)$ which in turn gives $\beta(f(A^o))^o\subset \beta(f(A))^o$----------(1)
Since $f(A^o)$ is β-open in Y, $\beta(f(A^o))^o = f(A^o)$------------------(2)
combining (1) and (2) we have $f(A^o)\subset \beta(f(A))^o$ for every subset A of X.

Theorem 4.4: If $f:X\to Y$ is almost contra β-closed and A⊆X is closed, f(A) is τ_β-open in Y.

Proof: Let A⊂ X be r-closed and f: X→ Y is almost contra β-closed ⇒ $f(A^o)\subset \beta(f(A))^o$ ⇒ $f(A) \subset \beta(f(A))^o$, since $f(A) = f(A^o)$. But $\beta(f(A))^o\subset f(A)$. Combining we get $f(A) = \beta(f(A))^o$. Therefore f(A) is τ_β-open in Y.

Corollary 4.3: If $f:X\to Y$ is almost contra r-closed, then f(A) is τ_β-open in Y if A is r-closed set in X.

Proof: Let A⊂ X be r-closed and f: X→ Y is almost contra r-closed ⇒ $f(A^o)\subset r(f(A))^o$ ⇒ $f(A^o)\subset \beta(f(A))^o$ (by theorem 4.3) ⇒ $f(A) \subset \beta(f(A))^o$, since $f(A) = f(A^o)$. But $\beta(f(A))^o\subset f(A)$. Combining we get $f(A) = \beta(f(A))^o$. Hence f(A) is τ_β-open in Y.

Theorem 4.5: $f:X\to Y$ is almost contra β-closed iff for each subset S of Y and each r-open set U containing $f^{-1}(S)$, there is an β-closed set V of Y such that S⊆ V and $f^{-1}(V) \subseteq U$.

Remark 7: Composition of two almost contra β-closed maps is not almost contra β-closed in general.

Theorem 4.6: Let X, Y, Z be topological spaces and every β-open set is r-closed in Y. Then the composition of two almost contra β-closed[almost contra r-closed] maps is almost contra β-closed.

Proof: (a) Let $f:X\to Y$ and $g:Y\to Z$ be almost contra β-closed maps. Let A be any r-closed set in X ⇒ f(A) is β-open in Y ⇒ f (A) is r-closed in Y (by assumption) ⇒ $g(f(A)) = gof(A)$ is β-open in Z. Therefore $g\ o\ f$ is almost contra β-closed.

(b) Let $f:X\rightarrow Y$ and $g:Y\rightarrow Z$ be almost contra β-closed maps. Let A be any r-closed set in $X \Rightarrow f$(A) is r-open in $Y \Rightarrow f$(A) is β-open in $Y \Rightarrow f$(A) is r-closed in Y (by assumption) $\Rightarrow g(f$(A)) is r-open in $Z \Rightarrow gof$(A) is β-open in Z. Therefore gof is almost contra β-closed.

Theorem 4.7: Let X, Y, Z be topological spaces and Y is discrete topological space in Y. Then the composition of two almost contra β-closed[almost contra r-closed] maps is almost contra β-closed.

Theorem 4.8: If $f:X\rightarrow Y$ is g-closed, $g:Y\rightarrow Z$ is contra β-closed [contra r-closed] and Y is $T_{1/2}$ [r-$T_{1/2}$] then $g\ o\ f$ is almost contra β-closed.

Proof: (a) Let A be a r-closed set in X. Then f(A) is g-closed set in $Y \Rightarrow f$ (A) is closed in Y as Y is $T_{1/2} \Rightarrow g(f$(A)) = gof(A) is β-open in Z since g is contra β-closed. Hence $g\ o\ f$ is almost contra β-closed.

(b) Let A be a r-closed set in X. Then f(A) is g-closed set in $Y \Rightarrow f$ (A) is closed in Y as Y is $T_{1/2} \Rightarrow g(f$(A)) is r-open in Z since g is contra r-closed $\Rightarrow gof$(A) is β-open in Z. Hence gof is almost contra β-closed.

Theorem 4.9: If $f:X\rightarrow Y$ is rg-open, $g:Y\rightarrow Z$ is contra β-closed[contra r-closed] and Y is r-$T_{1/2}$, then $g\ o\ f$ is almost contra β-closed.

Proof: Let A be a r-closed set in X. Then f(A) is rg-closed in $Y \Rightarrow f$(A) is r-closed in Y since Y is r-$T_{1/2} \Rightarrow$ g(f(A)) = go$f$(A) is β-open in Z. Hence gof is almost contra β-closed.

Corollary 4.4:
(i) If $f:X\rightarrow Y$ is g-closed, $g:Y\rightarrow Z$ is contra-closed[contra r-closed, contra semi-closed, contra pre-closed] and Y is $T_{1/2}$ [r-$T_{1/2}$] then $g\ o\ f$ is almost contra β-closed.
(ii) If $f:X\rightarrow Y$ is rg-open, $g:Y\rightarrow Z$ is contra-closed [contra r-closed, contra semi-closed, contra pre-closed] and Y is r-$T_{1/2}$, then $g\ o\ f$ is almost contra β-closed.

Theorem 4.10: If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that gof is almost contra β-closed [almost contra r-closed] then the following statements are true.
i) If f is continuous [r-continuous] and surjective then g is almost contra β-closed.
ii) If f is g-continuous, surjective and X is $T_{1/2}$ then g is almost contra β-closed.
iii) If f is rg-continuous, surjective and X is r-$T_{1/2}$ then g is almost contra β-closed.

Proof: (a) Let A be a r-closed set in $Y \Rightarrow f^{-1}$(A) is closed in $X \Rightarrow (g\ o\ f)(f^{-1}$(A)) = g(A) is β-open in Z. Hence g is almost contra β-closed.

(b) Let A be a r-closed set in Y. $\Rightarrow f^{-1}$(A) is g-closed in $X \Rightarrow f^{-1}$(A) is closed in X[since X is $T_{1/2}$] $\Rightarrow (g\ o\ f)(f^{-1}$(A)) = g(A) is β-open in Z. Hence g is almost contra β-closed.

(c) Let A be a r-closed set in $Y \Rightarrow f^{-1}$(A) is g-closed in $X \Rightarrow f^{-1}$(A) is closed in X[since X is r-$T_{1/2}$] $\Rightarrow (g\ o\ f)(f^{-1}$(A)) = g(A) is β-open in Z. Hence g is almost contra β-closed.

Theorem 4.11: If $f:X\rightarrow Y$ is almost contra β-closed and A is an closed set of X then $f_A:(X, \tau(A)) \rightarrow (Y, \sigma)$ is almost contra β-closed.

Proof: (a) Let F be a r-closed set in A. Then F = A∩E for some closed set E of X and so F is closed in $X \Rightarrow f$(F) is β-open in Y. But f(F) = f_A(F). Therefore f_A is almost contra β-closed.

Theorem 4.12: If $f:X\rightarrow Y$ is almost contra β-closed, X is $T_{1/2}$ and A is g-closed set of X then $f_A:(X, \tau(A)) \rightarrow (Y, \sigma)$ is almost contra β-closed.

Proof: Let F be a r-closed set in A. Then F = A∩E for some closed set E of X and so F is closed in $X \Rightarrow f$(F) is β-open in Y. But f(F) = f_A(F). Therefore f_A is almost contra β-closed.

Corollary 4.5: If f:$X \rightarrow Y$ is almost contra r-closed
(i) and A is an closed set of X then f_A:(X, τ(A)) → (Y, σ) is almost contra β-closed.
(ii) X is $T_{1/2}$ and A is g-closed set of X then f_A:(X, τ(A)) → (Y, σ) is almost contra β-closed.

Theorem 4.13: If f_i: $X_i \rightarrow Y_i$ be almost contra β-closed [almost contra r-closed] for i =1, 2. Let f: $X_1 x X_2 \rightarrow Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then f:$X_1 x X_2 \rightarrow Y_1 x Y_2$ is almost contra β-closed.

Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is r-closed in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)$ x $f_2(U_2)$ is β-open set in $Y_1 x Y_2$. Hence f is almost contra β-closed.

Theorem 4.14: Let h:$X \rightarrow X_1 x X_2$ be almost contra β-closed. Let f_i:$X \rightarrow X_i$ be defined as $h(x)=(x_1,x_2)$ and $f_i(x) = x_i$. Then f_i: $X \rightarrow X_i$ is almost contra β-closed for i =1, 2.

Proof: Let U_1 be r-closed in X_1, then $U_1 x\ X_2$ is r-closed in $X_1 x\ X_2$, and $h(U_1 x\ X_2)$ is β-open in X. But $f_1(U_1) = h(U_1 x\ X_2)$, therefore f_1 is almost contra β-closed. Similarly we can show that f_2 is also almost contra β-closed and thus f_i: X → X_i is almost contra β-closed for i = 1, 2.

Conclusion

In this paper we introduced the concept of almost contra β-open mappings, studied their basic properties and the interrelationship between other open maps.

References

[1] Abd El-Monsef.M.E., El-Deeb.S.N., and Mahmoud.R.A., β-open sets and β-continuous mappings, Bull of the Faculty of Sci, Assiut University. A(12)(1)(1983) 77 – 90
[2] Asit Kumar Sen and Bhattcharya.P., On preclosed mappings, Bull.Cal.Math.Soc.,85, 409-412(1993).
[3] Baker.C.W., Contra-open functions and contra-closed functions, Math. Today (Ahmedabad)15(1997), 19–24.
[4] Balasubramanian.S., and Vyjayanthi.P.A.S., almost contra-open and almost contra-closed Mappings –.
[5] Balasubramanian.S., and Vyjayanthi.P.A.S.,almost contra-semi-open and almost contra-semi-closed Mappings –
[6] Balasubramanian.S., and Vyjayanthi.P.A.S.,almost contra-pre-open and almost contra-pre-closed Mappings –
[7] Caldas,M., and Baker,C.W., Contra Pre-semiopen Maps, Kyungpook Math.Journal, 40 (2000), 379 – 389.
[8] Cammaroto.F., and Noiri.T., Almost irresolute functions, Indian J. Pure Appl. Math., 20(5) (1989), 472-482.
[9] Di Maio. G., and Noiri.T, I. J. P. A. M., 18(3) (1987) 226-233.
[10] Dontchev.J., Mem.Fac.Sci.Kochi Univ.ser.A., Math., 16(1995), 35-48.
[11] Dunham.W., $T_{1/2}$ Spaces, Kyungpook Math. J.17(1977), 161-169.
[12] Long.P.E., and Herington.L.L., Basic Properties of Regular Closed Functions, Rend. Cir. Mat. Palermo, 27(1978), 20-28.
[13] Malghan.S.R., Generalized closed maps, J. Karnatak Univ. Sci., 27(1982), 82 -88.
[14] Mashhour.A.S., Abd El-Monsef.M.E., and El-Deep.S.N., on precontinuous and weak precontinuous mappings, Proc.Math.Phy.Soc.Egypt, 53(1982)47-53.
[15] Mashour.A.S., Hasanein.I.A., and El.Deep.S.N., α-continuous and α-open mappings, Acta Math. Hungar.,41(1983), 213-218.
[16] Navalagi.G.B., "Definition Bank" in General Topology.
[17] Noiri.T., A generalization of closed mappings, Atti. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., 54 (1973),412-415.
[18] Noiri.T., Almost αg-closed functions and separation axioms, Acta Math. Hungar. 82(3) (1999),193-205.
[19] Palaniappan.N., Studies on Regular-Generalized Closed Sets and Maps in Topological Spaces, Ph. D Thesis, Alagappa University, Karikudi, (1995).